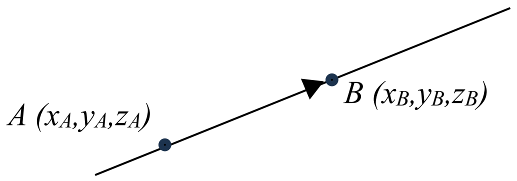
MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH
UNIVERSITY OF 20 AOÛT 1955 – SKIKDA

ACADEMIC YEAR 2023/2024 | DEPARTMENT OF COMPUTER SCIENCE
FACULTY OF SCIENCES

TUTORIAL SHEET N°1: VECTORS AND DERIVATIVES

1- Vectors



$$\overrightarrow{AB} = (x_B - x_A)\vec{\imath} + (y_B - y_A)\vec{\jmath} + (z_B - z_A)\vec{k}$$

$$\vec{V} = \overrightarrow{AB} = x_V\vec{\imath} + y_V\vec{\jmath} + z_V\vec{k} \implies \begin{cases} x_V = x_B - x_A \\ y_V = y_B - y_A \\ z_V = z_B - z_A \end{cases}$$

$\vec{V} + \vec{K} = (x_V + x_K)\vec{\imath} + (y_V + y_K)\vec{\jmath} + (z_V + z_K)\vec{k}$	Addition of vectors
$\vec{V} - \vec{K} = (x_V - x_K)\vec{\imath} + (y_V - y_K)\vec{\jmath} + (z_V - z_K)\vec{k}$	Subtraction of vectors
$\vec{V} \cdot \vec{K} = \|\vec{V}\| \cdot \|\vec{K}\| \cdot \cos\theta$ $\vec{V} \cdot \vec{K} = (x_V \cdot x_K) + (y_V \cdot y_K) + (z_V \cdot z_K)$	Dot product (scalar product) Analytical writing of Scalar product
$\lvert\vec{V} \wedge \vec{K}\rvert = \|\vec{V}\| \cdot \|\vec{K}\| \cdot \sin\theta$ $\vec{V} \wedge \vec{K} = (y_V z_K - z_V y_K)\vec{\imath} - (x_V z_K - z_V x_K)\vec{\jmath} + (x_V y_K - y_V x_K)\vec{k}$	Cross product (Vector product) Analytical writing of Vector product
$\vec{V} = \|\vec{V}\| \cdot \vec{U}$	Unit vector associated with each vector ($\vec{U}$)
$\vec{V} \parallel \vec{K} \implies \vec{V} \wedge \vec{K} = \vec{0}$ & $\vec{V} \perp \vec{K} \implies \vec{V} \cdot \vec{K} = \vec{0}$	Parallel and perpendicularity of two vectors

<u>Exercise 01</u>

The components of A(x_A,y_A,z_A), B(x_B,y_B,z_B) and C(x_C,y_C,z_C) are recorded in the corresponding table :

	x	y	z
A	1	-2	3
B	1	2	1
C	2	0	0

1- Write the vectors ($\overrightarrow{AB}$), ($\overrightarrow{AC}$) and ($\overrightarrow{BC}$), then represent them in the orthogonal basis (*Oxyz*)

2- Determine the magnitudes ($\|\overrightarrow{AB}\|$), ($\|\overrightarrow{AC}\|$) and ($\|\overrightarrow{BC}\|$)

3- What is the unit vector associated with each vector ?

4- Find the result of :

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{BC} = ?$$
$$\overrightarrow{AB} - \overrightarrow{BC} = ?$$
$$\overrightarrow{AB} \cdot \overrightarrow{AC} = ?$$
$$\overrightarrow{AB} \wedge \overrightarrow{AC} = ?$$
$$\overrightarrow{AB} \cdot \left(\overrightarrow{AC} \wedge \overrightarrow{BC}\right) = ?$$

5- Find the relationship between (x,y,z) components of $\vec{V}$ ($\vec{V} = x\,\vec{\imath} + y\,\vec{\jmath} + z\,\vec{k}$) when $\vec{V} \parallel \overrightarrow{AC}$

2- Derivative

$\frac{d\vec{V}}{dt} = \frac{\partial x_V}{\partial t}\vec{\imath} + \frac{\partial y_V}{\partial t}\vec{\jmath} + \frac{\partial z_V}{\partial t}\vec{k}$		Derivative of a vector with respect to time
$d(\vec{V}\cdot\vec{K}) = d(\vec{V})\cdot\vec{K} + \vec{V}\cdot d(\vec{K})$	$\frac{d}{dt}(\vec{V}\wedge\vec{K}) = \frac{d}{dt}(\vec{V})\wedge\vec{K} + \vec{V}\wedge\frac{d}{dt}(\vec{K})$	
$\frac{d\sin\theta}{dt} = \frac{d\theta}{dt}\cos\theta$	$\frac{d\cos\theta}{dt} = -\frac{d\theta}{dt}\sin\theta$	Derivatives of trigonometric and exponential Functions
$\frac{d(lnx)}{dt} = \frac{dx}{dt}\left(\frac{1}{x}\right)$	$\frac{d(e^x)}{dt} = \frac{dx}{dt}\cdot e^x$	
$cos^2\theta + sin^2\theta = 1$	$cos^2\theta = \frac{1+\cos 2\theta}{2}$	Useful relationships
$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ $\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$sin^2\theta = \frac{1-\cos 2\theta}{2}$	

Exercise 02

We have: $\begin{cases} \vec{V} = x\,\vec{\imath} - y\,\vec{\jmath} + z\,\vec{k} \\ \vec{K} = 2x\,\vec{\imath} + \frac{y}{4}\,\vec{\jmath} \end{cases}$

1- Find the derivative formula $\left(\frac{d\vec{V}}{dt}\right)$ and $\left(\frac{d\vec{K}}{dt}\right)$ when $(x = t^2 - 2), (y = -t^3)$ and $(z = t)$, then calculate the magnitude of the obtained result at $t = 3\ sec$

2- Determine $(\vec{V}\cdot\vec{K})$ and $(\vec{V}\wedge\vec{K})$, then calculate $\left(\frac{d(\vec{V}\cdot\vec{K})}{dt}\right)$ and $\left(\frac{d(\vec{V}\wedge\vec{K})}{dt}\right)$, when :

a. $x = t^2 - 2t,\ y = -t^3$ and $z = t$

b. $x = \sin 2t, y = 2\cos 2t$ and $z = 0$

c. $x = ln3t\ ,\ y = e^{2t}$ and $z = 0$

Exercise 03

The time equations of motion of a moving object are written as follows:

$$\begin{cases} x = t^2 - 2 \\ y = -t^2 \\ z = t \end{cases}$$

1- Determine the position vectors $\overrightarrow{OM}$, calculate its magnitude when $t = 2sec$ and then draw it in the homogeneous orthogonal coordinate reference (*Oxyz*)

2- Find the velocity $(\vec{V})$ and acceleration $(\vec{a})$ vectors. Calculate their magnitudes at $t = 2\ sec$