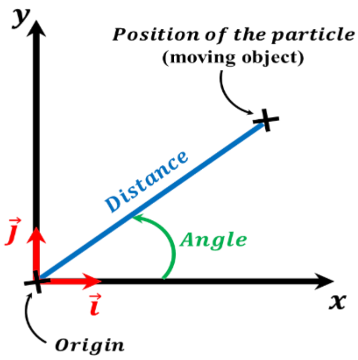
B. Polar Coordinate System:

Polar coordinates are a way of representing points in a two-dimensional system. Unlike the typical Cartesian coordinates (x, y), which use perpendicular axes, polar coordinates use a distance from a reference point (origin) and an angle relative to a reference direction.



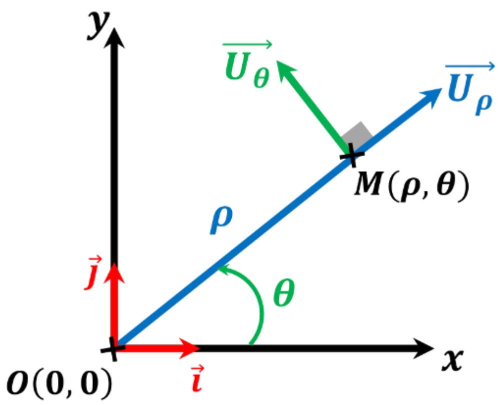
In polar coordinates, a point (moving object) is described by its distance from the origin (denoted by "ρ") and the angle formed by the line connecting the point and the origin with the reference direction (usually the positive x-axis), denoted by "θ" (theta).

The representation of a point in polar coordinates is written as (ρ, θ), where "ρ" is the radial distance and "θ" is the angle in standard position.

$(\overrightarrow{U_\rho}, \overrightarrow{U_\theta})$ represent the unit vectors in polar coordinates, similar to how $(\vec{i}, \vec{j})$ represent the unit vectors in cartesian coordinates.

i. $\overrightarrow{U_\rho}$: Associated with changes in the distance (ρ)

ii. $\overrightarrow{U_\theta}$: Related to changes in the angle (θ)

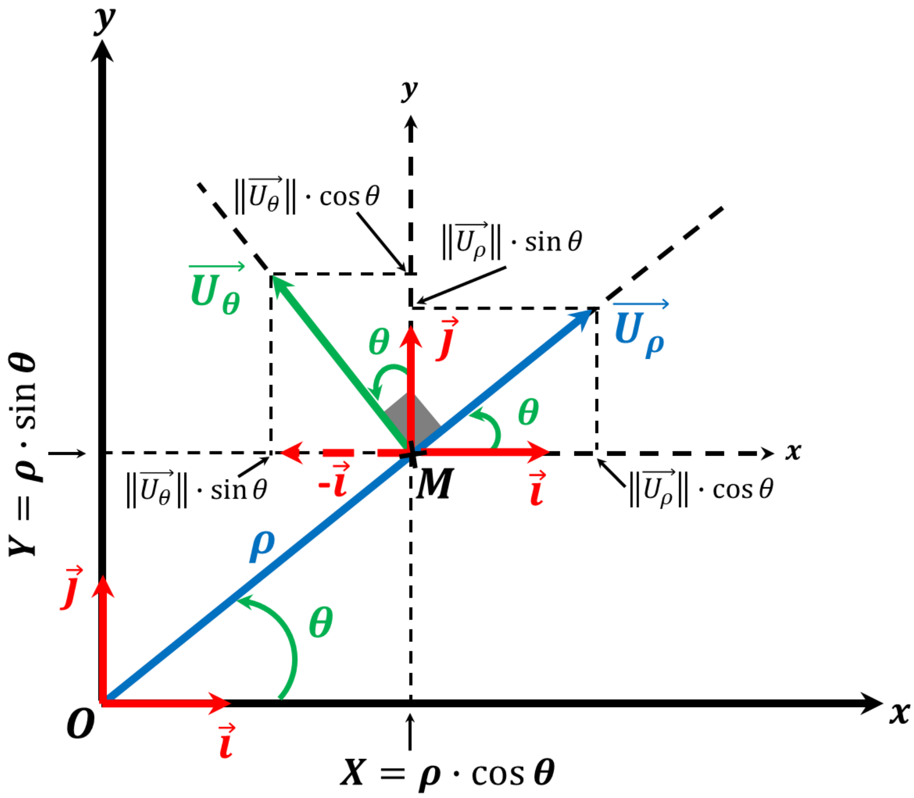


As mentioned above, it is evident that polar coordinates are linked to Cartesian coordinates. This is why a relationship between Cartesian coordinates (x, y) and polar coordinates (ρ, θ) can be expressed as follows:

$$x = \rho \cdot \cos\theta$$
$$y = \rho \cdot \sin\theta$$

Converting between polar and Cartesian coordinates requires utilizing trigonometric functions to establish the relationship between distance ρ and angle θ with the x and y coordinates.

This process works in both directions, allowing one to determine the distance and angle from given x and y coordinates or to find x and y coordinates from a given distance ρ and angle θ.



✓ Position vector $\overrightarrow{OM}$: The position vector in polar coordinates, like any vector, equals the product of its magnitude ($\|\overrightarrow{OM}\|$) and its associated unit vector ($\overrightarrow{U_\rho}$).

$$\overrightarrow{OM} = \|\overrightarrow{OM}\| \cdot \overrightarrow{U_\rho}$$

Given that the magnitude ($\|\overrightarrow{OM}\|$) corresponds to the distance (ρ), the polar position vector can be represented as follows:

$$\|\overrightarrow{OM}\| = \rho \ \Rightarrow \overrightarrow{OM} = \rho \cdot \overrightarrow{U_\rho}$$

Much like the relationship between cartesian coordinates (x, y) and polar coordinates (ρ, θ), a direct correlation between the unit vectors of both systems $(\vec{\imath}, \vec{\jmath})$ and $(\overrightarrow{U_\rho}, \overrightarrow{U_\theta})$ can be established by projecting $(\overrightarrow{U_\rho}, \overrightarrow{U_\theta})$ onto the *x* and *y* axes.

Projection onto *x*-axis:

$$\overrightarrow{U_\rho} = \|\overrightarrow{U_\rho}\| \cdot cos\,\theta\, \vec{\imath} + \|\overrightarrow{U_\rho}\| \cdot sin\,\theta\ \vec{\jmath}$$

Given that the magnitude of the polar unit vector, $\|\overrightarrow{U_\rho}\| = 1$, then:

$$\overrightarrow{U_\rho} = cos\,\theta\, \vec{\imath} + sin\,\theta\ \vec{\jmath}$$

Projection onto *y*-axis:

$$\overrightarrow{U_\theta} = \|\overrightarrow{U_\theta}\| \cdot sin\,\theta\ (-\vec{\imath}) + \|\overrightarrow{U_\theta}\| \cdot cos\,\theta\ \vec{\jmath}$$

$$\overrightarrow{U_\theta} = -\,\|\overrightarrow{U_\theta}\| \cdot \sin\theta\, \vec{\imath} + \|\overrightarrow{U_\theta}\| \cdot cos\,\theta\ \vec{\jmath}$$

Considering that the magnitude of the polar unit vector is $\|\overrightarrow{U_\theta}\| = 1$, then:

$$\overrightarrow{U_\theta} = -\,sin\,\theta\ \vec{\imath} + cos\,\theta\ \vec{\jmath}$$

- ✓ Velocity vector: The velocity vector $(\vec{V})$ is the derivative of the position vector $(\overrightarrow{OM})$ with respect to time (t).

$$\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \frac{d}{dt}(\rho.\overrightarrow{U_\rho})$$

$$\vec{V} = \frac{d}{dt}(\rho) \cdot \overrightarrow{U_\rho} + \rho \cdot \frac{d}{dt}(\overrightarrow{U_\rho}) = V_\rho \overrightarrow{U_\rho} + V_\theta\, \overrightarrow{U_\theta}$$

The solution of the derivative $(\frac{d}{dt}(\overrightarrow{U_\rho}))$ is presented below:

$$\frac{d}{dt}(\overrightarrow{U_\rho}) = \frac{d}{dt}(\cos\theta\ \vec{\imath} + \sin\theta\ \vec{\jmath})$$

$$\frac{d}{dt}(\overrightarrow{U_\rho}) = \frac{d}{dt} cos\,\theta)\ \vec{\imath} + \frac{d}{dt}(sin\,\theta)\ \vec{\jmath}$$

Given $\frac{d}{dt}\overrightarrow{U_\rho} = \dot{\overrightarrow{U_\rho}}$, therefore:

$$\frac{d}{dt}(\overrightarrow{U_\rho}) = \dot{\overrightarrow{U_\rho}} = (\frac{d}{dt}(cos\,\theta))\ \vec{\imath} + (\frac{d}{dt}(sin\,\theta))\ \vec{\jmath}$$

Generally, both ρ and θ change over time, which is why their derivatives can be expressed as follows:

$$\frac{d}{dt}(cos\,\theta) = \frac{d\theta}{d\theta} \times \frac{d\cos\theta}{dt} = \frac{d\theta}{dt} \times \frac{d\cos\theta}{d\theta}$$
$$\frac{d}{dt}(cos\,\theta) = \frac{d\theta}{dt} \times (-\sin\theta)$$

Considering $\frac{d\theta}{dt} = \dot{\theta}$, thus:

$$\frac{d}{dt}(cos\,\theta) = -\,\dot{\theta} \cdot \sin\theta$$

Similarly:

$$\frac{d}{dt}(sin\,\theta) = \frac{d\theta}{d\theta} \times \frac{d\,sin\,\theta}{dt} = \frac{d\theta}{dt} \times \frac{d\,sin\,\theta}{d\theta}$$
$$\frac{d}{dt}(sin\,\theta) = \frac{d\theta}{dt} \times (\cos\theta) \Rightarrow \frac{d}{dt}(sin\,\theta) = \dot{\theta} \cdot \cos\theta$$

So, $$\frac{d}{dt}\overrightarrow{U_\rho} = \dot{\overrightarrow{U_\rho}} = -\dot{\theta} \cdot \sin\theta\,\vec{\imath} + \dot{\theta} \cdot \cos\theta\,\vec{\jmath} = \dot{\theta}\underbrace{(-\sin\theta\,\vec{\imath} + \cos\theta\,\vec{\jmath})}_{=\,\overrightarrow{\boldsymbol{U_\theta}}}$$

Hence:

$$\frac{d}{dt}\overrightarrow{U_\rho} = \dot{\theta} \cdot \overrightarrow{U_\theta}$$

Finally, the velocity vector can be depicted as follows:

$$\vec{V} = \dot{\rho} \cdot \overrightarrow{U_\rho} + \rho \cdot \dot{\theta} \cdot \overrightarrow{U_\theta}$$

We deduce that the polar components of the velocity vector ($\vec{V}$) are :

$$V_\rho = \dot{\rho} \text{ and } V_\theta = \rho \cdot \dot{\theta}$$

✓ Acceleration vector: The acceleration vector ($\vec{a}$) is either the derivative of the velocity vector ($\vec{V}$) or the second derivative of the position vector ($\overrightarrow{OM}$) with respect to time (t):

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2\overrightarrow{OM}}{dt^2}$$

Upon substitution, the expression for the acceleration vector ($\vec{a}$) becomes:

$$\vec{a} = \frac{d^2\overrightarrow{OM}}{dt^2} = \frac{d^2}{dt^2}(\rho.\overrightarrow{U_\rho}) = \frac{d}{dt}\left(\underbrace{\frac{d}{dt}(\rho\cdot\overrightarrow{U_\rho})}_{=\vec{V}}\right)$$

The expression for acceleration ($\vec{a}$) can be developed by taking the derivative of the velocity components with respect to time as follows:

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\dot{\rho}\cdot\overrightarrow{U_\rho} + \rho\cdot\dot{\theta}\cdot\overrightarrow{U_\theta})$$

Expanding this derivative will provide the expression for the acceleration in polar coordinates:

$$\vec{a} = \frac{d}{dt}(\dot{\rho}\cdot\overrightarrow{U_\rho}) + \frac{d}{dt}(\rho\cdot\dot{\theta}\cdot\overrightarrow{U_\theta})$$

$$\vec{a} = \frac{d}{dt}(\dot{\rho})\cdot\overrightarrow{U_\rho} + \dot{\rho}\cdot\frac{d}{dt}(\overrightarrow{U_\rho}) + \frac{d}{dt}(\rho)\cdot\dot{\theta}\cdot\overrightarrow{U_\theta} + \rho\cdot\frac{d}{dt}(\dot{\theta})\cdot\overrightarrow{U_\theta} + \rho\cdot\dot{\theta}\cdot\frac{d}{dt}(\overrightarrow{U_\theta})$$

To expand on this, the derivative of the unit vector ($\overrightarrow{U_\theta}$) with respect to time can be developed as follows:

$$\frac{d}{dt}(\overrightarrow{U_\theta}) = \frac{d}{dt}(-\sin\theta\ \vec{\imath} + \cos\theta\,\vec{\jmath})$$

$$\frac{d}{dt}(\overrightarrow{U_\theta}) = \frac{d}{dt}(-sin\,\theta)\ \vec{\imath} + \frac{d}{dt}(\cos\theta)\vec{\jmath}$$

Given $\frac{d}{dt}\overrightarrow{U_\theta} = \dot{\overrightarrow{U_\theta}}$, hence:

$$\frac{d}{dt}(\overrightarrow{U_\theta}) = \dot{\overrightarrow{U_\theta}} = (\frac{d}{dt}(-\sin\theta))\ \vec{\imath} + (\frac{d}{dt}(cos\,\theta))\ \vec{\jmath}$$

Simplifying this formula by substituting the derivative of $\cos\theta$ and $\sin\theta$ with respect to time results in:

$$\frac{d}{dt}\overrightarrow{U_\theta} = \dot{\overrightarrow{U_\theta}} = -\dot{\theta}\cdot\cos\theta\,\vec{\imath} + \dot{\theta}\cdot(-\sin\theta)\,\vec{\jmath} = -\dot{\theta}(\underbrace{\sin\theta\,\vec{\imath} + \cos\theta\,\vec{\jmath}}_{=\overrightarrow{U_\rho}})$$

Therefore, the derivative of ($\overrightarrow{U_\theta}$) with respect to time is expressed as:

$$\frac{d}{dt}\overrightarrow{U_\theta} = -\dot{\theta}\cdot\overrightarrow{U_\rho}$$

The replacement of various terms in the acceleration vector ($\vec{a}$) is expressed as:

$$\vec{a} = \ddot{\rho} \cdot \overrightarrow{U_\rho} + \dot{\rho} \cdot \dot{\theta} \cdot \overrightarrow{U_\theta} + \rho \cdot \ddot{\theta} \cdot \overrightarrow{U_\theta} + \rho \cdot \dot{\theta} \cdot \left(- \dot{\theta} \cdot \overrightarrow{U_\rho}\right)$$

$$\vec{a} = \ddot{\rho} \cdot \overrightarrow{U_\rho} + \dot{\rho} \cdot \dot{\theta} \cdot \overrightarrow{U_\theta} + \dot{\rho} \cdot \dot{\theta} \cdot \overrightarrow{U_\theta} + \rho \cdot \ddot{\theta} \cdot \overrightarrow{U_\theta} - \rho \cdot \dot{\theta}^2 \cdot \overrightarrow{U_\rho}$$

Upon rearranging the previous relationship, the final formula for the acceleration vector ($\vec{a}$) in polar coordinates is expressed as:

$$\vec{a} = \left(\ddot{\rho} - \rho \cdot \dot{\theta}^2\right) \overrightarrow{U_\rho} + \left(\rho \cdot \ddot{\theta} + 2\, \dot{\rho} \cdot \dot{\theta}\right) \overrightarrow{U_\theta}$$

We deduce that the polar components of the acceleration vector ($\vec{a}$) are:

$$a_\rho = \ddot{\rho} - \rho \cdot \dot{\theta}^2 \text{ and } a_\theta = \rho \cdot \ddot{\theta} + 2\, \dot{\rho} \cdot \dot{\theta}$$

Exercise:

Given a particle moving in the plane with position defined in polar coordinates as $P(\rho, \theta)$, where ρ represents the radial distance from the origin and θ is the angle with respect to a reference axis. Given $\rho = 2t$ and $\theta = \frac{\pi}{3} t$.

1. Express the position vector $\overrightarrow{OM}$ of the particle in terms of Cartesian coordinates (x, y).
2. Determine the velocity vector $\vec{V}$ of the particle in terms of the unit vectors $\overrightarrow{U_\rho}$ and $\overrightarrow{U_\theta}$
3. Find the expression for the acceleration vector $\vec{a}$ in polar coordinates.

Solution

1. The position vector ($\overrightarrow{OM}$) expressed in terms of Cartesian coordinates x and y using polar coordinates ρ and θ is as follows:

The position vector is defined in Cartesian coordinate as follows: $\overrightarrow{OM} = x\,\vec{\imath} + y\,\vec{\jmath}$

Given: $x = \rho \cdot \cos\theta$ and $y = \rho \cdot sin\,\theta$

Therefore: $\overrightarrow{OM} = \rho \cdot \cos\theta\ \vec{\imath} + \rho \cdot \sin\theta\ \vec{\jmath}$

By substituting the polar coordinate values $\rho = 2t$ *and* $\theta = \frac{\pi}{3} t$, into the expression for the position vector, the result is:

$$\overrightarrow{OM} = 2t \cdot \cos\left(\frac{\pi}{3} t\right) \vec{\imath} + 2t \cdot \sin\left(\frac{\pi}{3} t\right) \vec{\jmath}$$

2. The expression for the velocity vector ($\vec{V}$) in terms of polar coordinates ρ and θ is developed through differentiation as follows:

Initially, the velocity is represented as:

$$\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \frac{d}{dt}\left(2t \cdot \cos\frac{\pi}{3}t\,\overrightarrow{U_\rho} + 2t \cdot \sin\frac{\pi}{3}t\,\overrightarrow{U_\theta}\right)$$

$$\vec{V} = \frac{d}{dt}\left(2t \cdot \cos\frac{\pi}{3}t\,\overrightarrow{U_\rho}\right) + \frac{d}{dt}\left(2t \cdot \sin\frac{\pi}{3}t\,\overrightarrow{U_\theta}\right)$$

Expanding the differentiation according to the derivative rules and applying the given derivatives related to t and the unit vectors, we derive:

$$\vec{V} = \frac{d}{dt}(2t) \cdot cos\left(\frac{\pi}{3}t\right)\overrightarrow{U_\rho} + 2t \cdot \frac{d}{dt}\left(cos\left(\frac{\pi}{3}t\right)\right)\overrightarrow{U_\rho} + 2t \cdot cos\left(\frac{\pi}{3}t\right)\frac{d}{dt}\left(\overrightarrow{U_\rho}\right)$$
$$+ \frac{d}{dt}(2t) \cdot \sin\left(\frac{\pi}{3}t\right)\overrightarrow{U_\theta} + 2t \cdot \frac{d}{dt}\left(sin\left(\frac{\pi}{3}t\right)\right)\overrightarrow{U_\theta} + 2t \cdot sin\left(\frac{\pi}{3}t\right)\frac{d}{dt}\left(\overrightarrow{U_\theta}\right)$$

Let's replace the given derivatives and relations involving the unit vectors into the previously derived expression for the velocity vector $\left(\vec{V}\right)$ to simplify it accordingly.

Given:

$$\frac{d}{dt}(2t) = 2$$

$$\frac{d}{dt}\left(cos\left(\frac{\pi}{3}t\right)\right) = -\frac{\pi}{3}\,sin\left(\frac{\pi}{3}t\right)$$

$$\frac{d}{dt}\left(sin\left(\frac{\pi}{3}t\right)\right) = \frac{\pi}{3}\,cos\left(\frac{\pi}{3}t\right)$$

$$\frac{d}{dt}\left(\overrightarrow{U_\rho}\right) = \dot{\theta}\,\overrightarrow{U_\theta} \text{ and } \frac{d}{dt}\left(\overrightarrow{U_\theta}\right) = -\,\dot{\theta}\,\overrightarrow{U_\rho}$$

Now let's substitute these derivatives and relations into the expression for the velocity vector ($\vec{V}$) to simplify it:

$$\vec{V} = 2 \cdot cos\left(\frac{\pi}{3}t\right)\overrightarrow{U_\rho} + 2t \cdot \left(-\frac{\pi}{3}\,sin\left(\frac{\pi}{3}t\right)\right)\overrightarrow{U_\rho} + 2t \cdot cos\left(\frac{\pi}{3}t\right) \cdot \left(\dot{\theta}\,\overrightarrow{U_\theta}\right)$$
$$+ 2 \cdot \sin\left(\frac{\pi}{3}t\right)\overrightarrow{U_\theta} + 2t \cdot \left(\frac{\pi}{3}\,cos\left(\frac{\pi}{3}t\right)\right)\overrightarrow{U_\theta} + 2t \cdot sin\left(\frac{\pi}{3}t\right) \cdot \left(-\,\dot{\theta}\,\overrightarrow{U_\rho}\right)$$

The rearranged formula for the velocity vector ($\vec{V}$) becomes:

$$\vec{V} = \left[2 \cdot cos\left(\frac{\pi}{3}t\right) - 2\,\dot{\theta}\,t \cdot sin\left(\frac{\pi}{3}t\right) - \frac{2\pi}{3}t \cdot sin\left(\frac{\pi}{3}t\right)\right]\overrightarrow{U_\rho}$$
$$+\left[2 \cdot \sin\left(\frac{\pi}{3}t\right) + 2\,\dot{\theta}\,t \cdot cos\left(\frac{\pi}{3}t\right) + \frac{2\pi}{3}t \cdot cos\left(\frac{\pi}{3}t\right)\right]\overrightarrow{U_\theta}$$

3. The expression for the acceleration vector ($\vec{a}$) in terms of polar coordinates ρ and θ is developed through differentiation as follows:

Initially, the acceleration is represented as:

$$\vec{a} = \frac{d^2\overrightarrow{OM}}{dt^2} = \frac{d^2}{dt^2}\left(2t \cdot \cos\frac{\pi}{3}t\,\overrightarrow{U_\rho} + 2t \cdot \sin\frac{\pi}{3}t\,\overrightarrow{U_\theta}\right)$$

$$\vec{a} = \frac{d^2}{dt^2}\left(2t \cdot \cos\frac{\pi}{3}t\,\overrightarrow{U_\rho}\right) + \frac{d^2}{dt^2}\left(2t \cdot \sin\frac{\pi}{3}t\,\overrightarrow{U_\theta}\right)$$

Expanding the differentiation according to the derivative rules and applying the given derivatives related to t and the unit vectors, we derive:

$$\vec{a} = \frac{d^2}{dt^2}(2t) \cdot cos\left(\frac{\pi}{3}t\right)\,\overrightarrow{U_\rho} + 2t \cdot \frac{d^2}{dt^2}\left(cos\left(\frac{\pi}{3}t\right)\right)\,\overrightarrow{U_\rho} + 2t \cdot cos\left(\frac{\pi}{3}t\right)\frac{d^2}{dt^2}\left(\overrightarrow{U_\rho}\right)$$
$$+\frac{d^2}{dt^2}(2t) \cdot \sin\left(\frac{\pi}{3}t\right)\,\overrightarrow{U_\theta} + 2t \cdot \frac{d^2}{dt^2}\left(sin\left(\frac{\pi}{3}t\right)\right)\,\overrightarrow{U_\theta} + 2t \cdot sin\left(\frac{\pi}{3}t\right)\frac{d^2}{dt^2}\left(\overrightarrow{U_\theta}\right)$$

Let's replace the given derivatives and relations involving the unit vectors into the previously derived expression for the acceleration vector ($\vec{a}$) to simplify it accordingly.

Given:

$$\frac{d^2}{dt^2}(2t) = 0$$

$$\frac{d^2}{dt^2}\left(cos\left(\frac{\pi}{3}t\right)\right) = -\left(\frac{\pi}{3}\right)^2 cos\left(\frac{\pi}{3}t\right)$$

$$\frac{d^2}{dt^2}\left(sin\left(\frac{\pi}{3}t\right)\right) = -\left(\frac{\pi}{3}\right)^2 sin\left(\frac{\pi}{3}t\right)$$

$$\frac{d^2}{dt^2}\left(\overrightarrow{U_\rho}\right) = \ddot{\theta}\,\overrightarrow{U_\theta} - \dot{\theta}^2\overrightarrow{U_\rho} \text{ and } \frac{d^2}{dt^2}\left(\overrightarrow{U_\theta}\right) = -\ddot{\theta}\,\overrightarrow{U_\rho} + \dot{\theta}^2\overrightarrow{U_\theta}$$

Now let's substitute these derivatives and relations into the expression for the acceleration vector ($\vec{V}$) to simplify it:

$$\vec{a} = 2t \cdot \left(-\left(\frac{\pi}{3}\right)^2 cos\left(\frac{\pi}{3}t\right)\right) \overrightarrow{U_\rho} + 2t \cdot cos\left(\frac{\pi}{3}t\right) \cdot \left(\ddot{\theta}\,\overrightarrow{U_\theta} - \dot{\theta}^2\overrightarrow{U_\rho}\right)$$
$$+ 2t \cdot \left(-\left(\frac{\pi}{3}\right)^2 sin\left(\frac{\pi}{3}t\right)\right) \overrightarrow{U_\theta} + 2t \cdot sin\left(\frac{\pi}{3}t\right) \cdot \left(-\ddot{\theta}\,\overrightarrow{U_\rho} + \dot{\theta}^2\overrightarrow{U_\theta}\right)$$

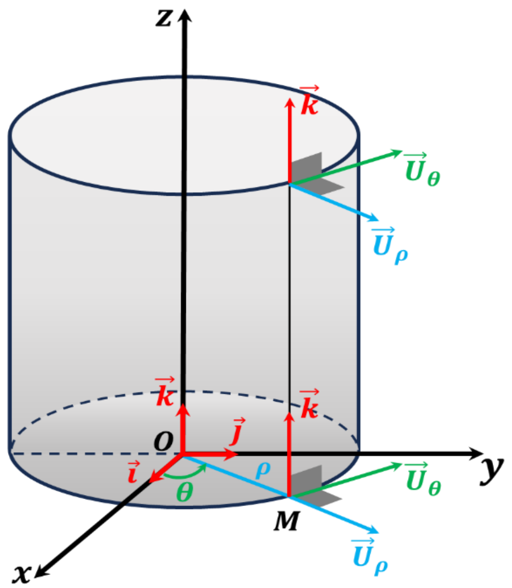
The rearranged formula for the acceleration vector ($\vec{a}$) becomes:

$$\vec{a} = -\left[2\,\ddot{\theta}\,t \cdot sin\left(\frac{\pi}{3}t\right) + 2t \cdot \left(\frac{\pi}{3}\right)^2 \cdot cos\left(\frac{\pi}{3}t\right) + 2\,\dot{\theta}^2\,t \cdot cos\left(\frac{\pi}{3}t\right)\right]\overrightarrow{U_\rho}$$
$$+\left[2\,\ddot{\theta}\,t \cdot cos\left(\frac{\pi}{3}t\right) - 2t \cdot \left(\frac{\pi}{3}\right)^2 sin\left(\frac{\pi}{3}t\right) + 2\,\dot{\theta}^2\,t \cdot sin\left(\frac{\pi}{3}t\right)\right]\overrightarrow{U_\theta}$$

C. Cylindrical coordinate systems

Actually, the cylindrical coordinate system is an extension of the 2D polar coordinate system into 3D space. It involves specifying a point in space using three components (ρ, θ, z). The three components in cylindrical coordinates are:

1. ρ: The radial distance from the z-axis to the point in the xy-plane.
2. θ: The angle measured counterclockwise from the x-axis to the projection of the point onto the (x, y)-plane.
3. z: The height or vertical position above (or below) the (x, y)-plane.



Within the cylindrical coordinate system, we can define position, velocity, and acceleration similar to other coordinate systems, though with some variations in terms of the variables used and their transformation.

✓ Position vector ($\overrightarrow{OM}$)

The position vector in cylindrical coordinates, $\left(\overrightarrow{OM_{Cyl.}}\right)$, is defined as the sum of the position vector in polar coordinates, $\left(\overrightarrow{OM_{Pol.}}\right)$, and the vertical component $\left(z\,\vec{k}\right)$.
This can be described as:

$$\overrightarrow{OM_{Cyl.}} = \overrightarrow{OM_{Pol.}} + z\,\vec{k}$$

In vector form, this relationship is written as:

$$\overrightarrow{OM} = \rho\,\overrightarrow{U_\rho} + z\,\vec{k}$$

Where:

- ρ: is the radial distance from the z-axis to the point in the (x, y)-plane.
- z: is the vertical position above (or below) the (x, y)-plane.
- $\overrightarrow{U_\rho}$: is the unit vector in the radial direction.
- $\vec{k}$: is the unit vector in the vertical direction.

So, the position vector in cylindrical coordinates can be expressed as the sum of the position vector in polar coordinates and the vertical component $\left(z\,\vec{k}\right)$.

✓ Velocity vector ($\vec{V}$)

The velocity vector in cylindrical coordinates, denoted as $\left(\overrightarrow{V_{Cyl.}}\right)$, is a combination of the velocity vector in polar coordinates, $\left(\overrightarrow{V_{Pol.}}\right)$, and the first derivative (rate of change) of the vertical component ($\dot{z}\,\vec{k}$).

$$\overrightarrow{V_{Cyl.}} = \frac{d}{dt}\left(\overrightarrow{OM_{Cyl.}}\right) = \frac{d}{dt}\left[\overrightarrow{OM_{Pol.}} + z\,\vec{k}\right] = \underbrace{\frac{d}{dt}\left(\overrightarrow{OM_{Pol.}}\right)}_{\left(\overrightarrow{V_{Pol.}}\right)} + \underbrace{\left(\frac{dz}{dt}\right)}_{(\dot{z})}\vec{k}$$

The velocity in cylindrical coordinates can indeed be expressed as:

$$\overrightarrow{V_{Cyl.}} = \overrightarrow{V_{Pol.}} + \dot{z}\,\vec{k}$$

With the representation of the velocity vector in polar coordinates as:

$$\overrightarrow{V_{Pol.}} = \dot{\rho}\cdot\overrightarrow{U_\rho} + \rho\cdot\dot{\theta}\cdot\overrightarrow{U_\theta}$$

The transformation to the velocity vector in cylindrical coordinates is accurately described as:

$$\overrightarrow{V_{Cyl.}} = \dot{\rho} \cdot \overrightarrow{U_\rho} + \rho \cdot \dot{\theta} \cdot \overrightarrow{U_\theta} + \dot{z}\,\vec{k}$$

Where $\dot{\rho}, \dot{\theta}$ and $\dot{z}$ are the rates of change of ρ, θ, and z with respect to time, respectively.

- ✓ Acceleration vector ($\vec{a}$)

The acceleration vector in cylindrical coordinates, denoted as $\left(\overrightarrow{a_{Cyl.}}\right)$, is a combination of the acceleration vector in polar coordinates, $\left(\overrightarrow{a_{Pol.}}\right)$, and the second derivative (rate of change) of the vertical component ($\ddot{z}\,\vec{k}$). Two relationships can

The calculation of the acceleration in cylindrical coordinates $\left(\overrightarrow{a_{Cyl.}}\right)$ from position $\left(\overrightarrow{OM_{Cyl.}}\right)$ involves the second derivative of the position vector with respect to time. It's correctly derived as:

$$\overrightarrow{a_{Cyl.}} = \frac{d^2}{dt^2}\left(\overrightarrow{OM_{Cyl.}}\right) = \frac{d^2}{dt^2}\left[\overrightarrow{OM_{Pol.}} + z\,\vec{k}\right] = \underbrace{\frac{d^2}{dt^2}\left(\overrightarrow{OM_{Pol.}}\right)}_{=\left(\overrightarrow{a_{Pol.}}\right)} + \underbrace{\left(\frac{d^2 z}{dt^2}\right)}_{=(\ddot{z})}\vec{k}$$

The acceleration in cylindrical coordinates $\left(\overrightarrow{a_{Cyl.}}\right)$ is also derived from the derivative of the velocity in cylindrical coordinates $\left(\overrightarrow{V_{Cyl.}}\right)$. It is accurately represented as:

$$\overrightarrow{a_{Cyl.}} = \frac{d}{dt}\left(\overrightarrow{V_{Cyl.}}\right) = \frac{d}{dt}\left[\overrightarrow{V_{Pol.}} + \dot{z}\,\vec{k}\right] = \underbrace{\frac{d}{dt}\left(\overrightarrow{V_{Pol.}}\right)}_{=\left(\overrightarrow{a_{Pol.}}\right)} + \underbrace{\left(\frac{d\dot{z}}{dt}\right)}_{=(\ddot{z})}\vec{k}$$

The acceleration in cylindrical coordinates can indeed be expressed as:

$$\overrightarrow{a_{Cyl.}} = \overrightarrow{a_{Pol.}} + \ddot{z}\,\vec{k}$$

With the representation of the velocity vector in polar coordinates as:

$$\overrightarrow{a_{Pol.}} = \left(\ddot{\rho} - \rho \cdot \dot{\theta}^2\right)\overrightarrow{U_\rho} + \left(\rho \cdot \ddot{\theta} + 2\,\dot{\rho} \cdot \dot{\theta}\right)\overrightarrow{U_\theta}$$

The transformation to the acceleration vector in cylindrical coordinates is accurately described as:

$$\overrightarrow{a_{Cyl.}} = \left(\ddot{\rho} - \rho \cdot \dot{\theta}^2\right)\overrightarrow{U_\rho} + \left(\rho \cdot \ddot{\theta} + 2\,\dot{\rho} \cdot \dot{\theta}\right)\overrightarrow{U_\theta} + \ddot{z}\,\vec{k}$$